MATH 2204 **Sample Exam 2/Study Guide**

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Name:

Instructions:

- All answers must be written clearly.
- You may use a calculator (TI-84 or below), but you must show all your work in order to receive credit. This includes any multiple choice questions! No credit will be given to any problem unless work is shown.
- Be sure to erase or cross out any work that you do not want graded.
- If two answers are circled in the multiple choice, then zero credit is given.
- If you need extra space, you may use the back sides of the exam pages (if you do, please write me a note so that I know where to look).
- Any cheating will result in an immediate F in the course.
- Partial credit will be given to open ended problems.

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	0	0	0	0	0	0	0	0	0	0	0	0
Score:												

1. Study HW Problems 1-5 in Section 3.1 - 2nd Order Linear Homogeneous constant coefficients - Real distinct roots
   - For more Sample Problems with solutions:
     - Click here: http://tutorial.math.lamar.edu/Classes/DE/RealRoots.aspx
2. Study HW Problems 1-4 in Section 3.2 - General Solution Theorem - the Wronskian
3. Study HW Problems 1-2 in Section 3.3 - 2nd Order Linear Homogeneous constant coefficients - complex roots
   - For more Sample Problems with solutions:
     - Click here: http://tutorial.math.lamar.edu/Classes/DE/ComplexRoots.aspx
4. Study HW Problems 1-3 in Section 3.4.1 - 2nd Order Linear Homogeneous constant coefficients - Real repeated roots
   - For more Sample Problems with solutions:
     - Click here: http://tutorial.math.lamar.edu/Classes/DE/RepeatedRoots.aspx
5. Study HW Problems 1-4 in Section 3.4.2 - Method of Reduction of Order
   - For more Sample Problems with solutions:
     - Click here: http://tutorial.math.lamar.edu/Classes/DE/ReductionofOrder.aspx
6. Study HW Problems 1-7 in Section 3.5 - non-homogeneous equations - MOUC
   - For more Sample Problems with solutions:
     - Click here: http://tutorial.math.lamar.edu/Classes/DE/NonhomogeneousDE.aspx
7. Study HW Problems 1-4 in Section 3.6 - non-homogeneous equations - Variation of Parameters
   - For more Sample Problems with solutions:
     - Click here: http://tutorial.math.lamar.edu/Classes/DE/VariationofParameters.aspx
8. Study HW Problems 1-4 in Section 3.7 - Mechanical and Electrical Vibrations
   - For more Sample Problems with solutions:
     - http://tutorial.math.lamar.edu/Classes/DE/Vibrations.aspx
9. Study HW Problems 1-6 in Section 4.1 - Higher Order Systems - homogeneous
   - For more Sample Problems with solutions:
     - http://tutorial.math.lamar.edu/Classes/DE/HOHomogeneousDE.aspx
10. Study HW Problems 1-4 in Section 4.2 - Higher Order Systems - non-homogeneous
    - For more Sample Problems with solutions:
      - http://tutorial.math.lamar.edu/Classes/DE/HOUndeterminedCoeff.aspx
11. Study only HW Problem 3 in Section 6.1 - Intro to Laplace Transforms
    - For more Sample Problems with solutions:
      - http://tutorial.math.lamar.edu/Classes/DE/LaplaceTransforms.aspx

Formula Sheet

- **General Solution Theorem for Homogeneous Equations:**

Theorem 1 ***(General Solution Theorem)*** *Suppose y_1 and y_2 are two solutions to the ODE*

$$y'' + p(t)y' + q(t)y = 0$$

in some interval I, where p, q are continuous. Then the family of solutions

$$y(t) = c_1 y_1(t) + c_2 y_2(t)$$

*for arbtitrary c_1, c_2 is the **general solution** (meaning includes every solution to the ODE) if and only if the Wronskian $W(y_1, y_2)$ is not zero for at least one point t_0 in I.*

- **Variation of Parameters:**

Theorem 2 *(Variation of Parameters) If $p, q,$ and g are continuous on an open interval I, and if the functions $\{y_1, y_2\}$ form a fundamental set of solutions to the corresponding homogeneous EQ*

$$y'' + p(t)y' + q(t)y = 0.$$

Then a particular solution to

$$y'' + p(t)y' + q(t)y = g(t),$$

is given by

$$\begin{aligned} y_p(t) &= -y_1(t) \int_{t_0}^{t} \frac{y_2(s)g(s)}{W(y_1, y_2)(s)} ds + y_2(t) \int_{t_0}^{t} \frac{y_1(s)g(s)}{W(y_1, y_2)(s)} ds \\ &= -y_1(t) \left[\int \frac{y_2(t)g(t)}{W(y_1, y_2)(t)} dt \right] + y_2(t) \left[\int \frac{y_1(t)g(t)}{W(y_1, y_2)(t)} dt \right], \text{ if an antiderivative exists} \end{aligned}$$

*where t_0 is any value in I. Then **general solution** to the non-homogeneous solution is*

$$y(t) = c_1 y_1(t) + c_2 y_2(t) + y_p(t).$$

- **Motion of a spring:**
- The motion of $u(t)$ is modeled by the following:

$$mu''(t) + \gamma u'(t) + ku(t) = F(t) \quad u(0) = u_0, u'(0) = v_0.$$

  where m, γ, k are positive.

  - m is found from $w = mg$
  - γ is given in units of $\frac{\text{weight unit}\cdot s}{\text{distance unit}}$.
  - k is found using Hooke's Law, $mg = kL$

- **(Existence and Uniqueness Theorem for n Order Linear ODEs):** If the functions $p_1, \ldots, p_{(n-1}$ and g are continuous on an open interval $I = (a, b)$ containing the point $t = t_0$, then there **exists** a **unique** function $y = \phi(t)$ that satisfies the IVP

$$y^{(n)} + p_{n-1}(t)y^{(n-1)} + \cdots + p_1(t)y' + p_0(t)y = g(t). \quad (\star)$$

  with initial conditions

$$y(t_0) = y_0,\ y'(t_0) = y_0',\ \cdots, y^{(n-1)}(t_0) = y_0^{(n-1)}. \quad (\star)$$

  for each t in I.

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \ c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$